# Sage Quick Reference

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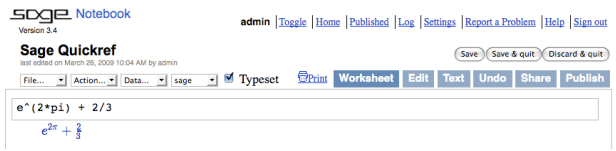
## Notebook




Evaluate cell: ⟨shift-enter⟩
Evaluate cell creating new cell: ⟨alt-enter⟩
Split cell: ⟨control-;⟩
Join cells: ⟨control-backspace⟩
Insert math cell: click blue line between cells
Insert text/HTML cell: shift-click blue line between cells
Delete cell: delete content then backspace

## Command line

*com*⟨tab⟩ complete *command*
`*`*bar*`*?` list command names containing "*bar*"
*command*`?`⟨tab⟩ shows documentation
*command*`??`⟨tab⟩ shows source code
`a.`⟨tab⟩ shows methods for object `a` (more: `dir(a)`)
`a._`⟨tab⟩ shows hidden methods for object `a`
`search_doc("`*string or regexp*`")` fulltext search of docs
`search_src("`*string or regexp*`")` search source code
`_` is previous output

## Numbers

Integers: $\mathbb{Z}$ = `ZZ` e.g. `-2 -1 0 1 10^100`
Rationals: $\mathbb{Q}$ = `QQ` e.g. `1/2 1/1000 314/100 -2/1`
Reals: $\mathbb{R} \approx$ `RR` e.g. `.5 0.001 3.14 1.23e10000`
Complex: $\mathbb{C} \approx$ `CC` e.g. `CC(1,1) CC(2.5,-3)`
Double precision: `RDF` and `CDF` e.g. `CDF(2.1,3)`
Mod $n$: $\mathbb{Z}/n\mathbb{Z}$ = `Zmod` e.g. `Mod(2,3) Zmod(3)(2)`
Finite fields: $\mathbb{F}_q$ = `GF` e.g. `GF(3)(2) GF(9,"a").0`
Polynomials: $R[x,y]$ e.g. `S.<x,y>=QQ[] x+2*y^3`
Series: $R[[t]]$ e.g. `S.<t>=QQ[[]] 1/2+2*t+O(t^2)`
$p$-adic numbers: $\mathbb{Z}_p \approx$`Zp`, $\mathbf{Q}_p \approx$`Qp` e.g. `2+3*5+O(5^2)`
Algebraic closure: $\overline{\mathbb{Q}}$ = `QQbar` e.g. `QQbar(2^(1/5))`
Interval arithmetic: `RIF` e.g. `RIF((1,1.00001))`
Number field: `R.<x>=QQ[]; K.<a>=NumberField(x^3+x+1)`

## Arithmetic

$ab$ = `a*b` $\frac{a}{b}$ = `a/b` $a^b$ = `a^b` $\sqrt{x}$ = `sqrt(x)`
$\sqrt[n]{x}$ = `x^(1/n)` $|x|$ = `abs(x)` $\log_b(x)$ = `og(x,b)`

Sums: $\sum_{i=k}^{n} f(i)$ = `sum(f(i) for i in (k..n))`

Products: $\prod_{i=k}^{n} f(i)$ = `prod(f(i) for i in (k..n))`

## Constants and functions

Constants: $\pi$ = `pi` $e$ = `e` $i$ = `i` $\infty$ = `oo`
$\phi$ = `golden_ratio` $\gamma$ = `euler_gamma`
Approximate: `pi.n(digits=18)` = 3.14159265358979324
Functions: `sin cos tan sec csc cot sinh cosh tanh sech csch coth log ln exp` ...
Python function: `def f(x): return x^2`

## Interactive functions

Put `@interact` before function (vars determine controls)

```
@interact
def f(n=[0..4], s=(1..5), c=Color("red")):
    var("x")
    show(plot(sin(n+x^s),-pi,pi,color=c))
```

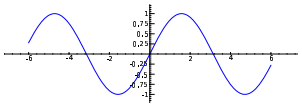
## Symbolic expressions

Define new symbolic variables: `var("t u v y z")`
Symbolic function: e.g. $f(x) = x^2$ `f(x)=x^2`
Relations: `f==g f<=g f>=g f<g f>g`
Solve $f = g$: `solve(f(x)==g(x), x)`
`solve([f(x,y)==0, g(x,y)==0], x,y)`
`factor(...)` `expand(...)` `(...).simplify_...`
`find_root(f(x), a, b)` find $x \in [a,b]$ s.t. $f(x) \approx 0$

## Calculus

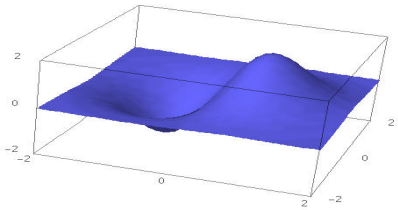
$\lim_{x \to a} f(x)$ = `limit(f(x), x=a)`
$\frac{d}{dx}(f(x))$ = `diff(f(x),x)`
$\frac{\partial}{\partial x}(f(x,y))$ = `diff(f(x,y),x)`
`diff` = `differentiate` = `derivative`
$\int f(x)dx$ = `integral(f(x),x)`
$\int_a^b f(x)dx$ = `integral(f(x),x,a,b)`
$\int_a^b f(x)dx \approx$ `numerical_integral(f(x),a,b)`
Taylor polynomial, deg $n$ about $a$: `taylor(f(x),x,a,n)`

## 2D graphics



`line([`$(x_1,y_1),\ldots,(x_n,y_n)$`],`*options*`)`
`polygon([`$(x_1,y_1),\ldots,(x_n,y_n)$`],`*options*`)`
`circle(`$(x,y)$`,`$r$`,`*options*`)`
`text("txt",`$(x,y)$`,`*options*`)`
*options* as in `plot.options`,
e.g. `thickness=`*pixel*, `rgbcolor=`$(r,g,b)$, `hue=`$h$
where $0 \le r,b,g,h \le 1$
`show(`*graphic*`,` *options*`)`
use `figsize=[w,h]` to adjust size
use `aspect_ratio=`*number* to adjust aspect ratio
`plot(f(`$x$`),`$(x, x_{\min}, x_{\max})$`,`*options*`)`
`parametric_plot((f(`$t$`),g(`$t$`)),`$(t, t_{\min}, t_{\max})$`,`*options*`)`
`polar_plot(f(`$t$`),`$(t, t_{\min}, t_{\max})$`,`*options*`)`
combine: `circle((1,1),1)+line([(0,0),(2,2)])`
`animate(`*list of graphics, options*`).show(delay=20)`

## 3D graphics

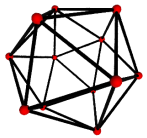


`line3d([`$(x_1,y_1,z_1),\ldots,(x_n,y_n,z_n)$`],`*options*`)`
`sphere(`$(x,y,z)$`,`$r$`,`*options*`)`
`text3d("txt",` $(x,y,z)$`,` *options*`)`
`tetrahedron(`$(x,y,z)$`,`*size*`,`*options*`)`
`cube(`$(x,y,z)$`,`*size*`,`*options*`)`
`octahedron(`$(x,y,z)$`,`*size*`,`*options*`)`
`dodecahedron(`$(x,y,z)$`,`*size*`,`*options*`)`
`icosahedron(`$(x,y,z)$`,`*size*`,`*options*`)`
`plot3d(f(`$x,y$`),`$(x, x_b, x_e)$`,` $(y, y_b, y_e)$`,`*options*`)`
`parametric_plot3d((f,g,h),`$(t, t_b, t_e)$`,`*options*`)`
`parametric_plot3d((f(`$u,v$`),g(`$u,v$`),h(`$u,v$`)),`
$(u, u_b, u_e), (v, v_b, v_e)$`,`*options*`)`
*options*: `aspect_ratio=[1,1,1]`, `color="red"`,
`opacity=0.5`, `figsize=6`, `viewer="tachyon"`

## Discrete math

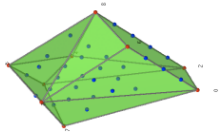
$\lfloor x \rfloor$ = `floor(x)` $\lceil x \rceil$ = `ceil(x)`
Remainder of $n$ divided by $k$ = `n%k` $k|n$ iff `n%k==0`
$n!$ = `factorial(n)` $\binom{x}{m}$ = `binomial(x,m)`
$\phi(n)$ = `euler_phi(`$n$`)`
Strings: e.g. `s = "Hello"` = `"He"+'llo'`
`s[0]="H"` `s[-1]="o"` `s[1:3]="el"` `s[3:]="lo"`
Lists: e.g. `[1,"Hello",x] = []+[1,"Hello"]+[x]`
Tuples: e.g. `(1,"Hello",x)` (immutable)
Sets: e.g. $\{1,2,1,a\}$ = `Set([1,2,1,"a"])` $(= \{1,2,a\})$
List comprehension $\approx$ set builder notation, e.g.
$\{f(x)|x \in X, x > 0\}$ =`Set([f(x) for x in X if x>0])`

## Graph theory



Graph: `G = Graph({0:[1,2,3], 2:[4]})`
Directed Graph: `DiGraph(`*dictionary*`)`
Graph families: `graphs.`⟨tab⟩
Invariants: `G.chromatic_polynomial()`, `G.is_planar()`
Paths: `G.shortest_path()`
Visualize: `G.plot()`, `G.plot3d()`
Automorphisms: `G.automorphism_group()`,
`G1.is_isomorphic(G2)`, `G1.is_subgraph(G2)`

## Combinatorics

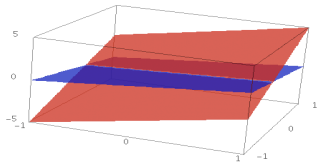


Integer sequences: `sloane_find(`*list*`)`, `sloane.`⟨tab⟩
Partitions: `P=Partitions(`$n$`)` `P.count()`
Combinations: `C=Combinations(`*list*`)` `C.list()`
Cartesian product: `CartesianProduct(P,C)`
Tableau: `Tableau([[1,2,3],[4,5]])`
Words: `W=Words("abc"); W("aabca")`
Posets: `Poset([[1,2],[4],[3],[4],[]])`
Root systems: `RootSystem(["A",3])`
Crystals: `CrystalOfTableaux(["A",3], shape=[3,2])`
Lattice Polytopes: `A=random_matrix(ZZ,3,6,x=7)`
`L=LatticePolytope(A) L.npoints() L.plot3d()`

## Matrix algebra

$\begin{pmatrix}1\\2\end{pmatrix}$ = `vector([1,2])`
$\begin{pmatrix}1&2\\3&4\end{pmatrix}$ = `matrix(QQ,[[1,2],[3,4]], sparse=False)`
$\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}$ = `matrix(QQ,2,3,[1,2,3, 4,5,6])`
$\begin{vmatrix}1&2\\3&4\end{vmatrix}$ = `det(matrix(QQ,[[1,2],[3,4]]))`
$Av$ = `A*v` $A^{-1}$ = `A^-1` $A^t$ = `A.transpose()`
Solve $Ax = v$: `A\v` or `A.solve_right(v)`
Solve $xA = v$: `A.solve_left(v)`
Reduced row echelon form: `A.echelon_form()`
Rank and nullity: `A.rank()` `A.nullity()`
Hessenberg form: `A.hessenberg_form()`
Characteristic polynomial: `A.charpoly()`
Eigenvalues: `A.eigenvalues()`
Eigenvectors: `A.eigenvectors_right()` (also left)
Gram-Schmidt: `A.gram_schmidt()`
Visualize: `A.plot()`
LLL reduction: `matrix(ZZ,...).LLL()`
Hermite form: `matrix(ZZ,...).hermite_form()`

## Linear algebra



Vector space $K^n$ = `K^n` e.g. `QQ^3 RR^2 CC^4`
Subspace: `span(vectors,` *field*`)`
E.g., `span([[1,2,3], [2,3,5]], QQ)`
Kernel: `A.right_kernel()` (also left)
Sum and intersection: `V + W` and `V.intersection(W)`
Basis: `V.basis()`
Basis matrix: `V.basis_matrix()`
Restrict matrix to subspace: `A.restrict(V)`
Vector in terms of basis: `V.coordinates(`*vector*`)`

## Numerical mathematics

Packages: `import numpy, scipy, cvxopt`
Minimization: `var("x y z")`
`minimize(x^2+x*y^3+(1-z)^2-1, [1,1,1])`

## Number theory

Primes: `prime_range(n,m)`, `is_prime`, `next_prime`
Factor: `factor(n)`, `qsieve(n)`, `ecm.factor(n)`
Kronecker symbol: $\left(\frac{a}{b}\right)$ = `kronecker_symbol(`$a$`,`$b$`)`
Continued fractions: `continued_fraction(x)`
Bernoulli numbers: `bernoulli(n)`, `bernoulli_mod_p(p)`
Elliptic curves: `EllipticCurve(`$[a_1, a_2, a_3, a_4, a_6]$`)`
Dirichlet characters: `DirichletGroup(`$N$`)`
Modular forms: `ModularForms(`*level*`,` *weight*`)`
Modular symbols: `ModularSymbols(`*level*`,`*weight*`,`*sign*`)`
Brandt modules: `BrandtModule(`*level*`,` *weight*`)`
Modular abelian varieties: `J0(`$N$`)`, `J1(`$N$`)`

## Group theory

`G = PermutationGroup([[(1,2,3),(4,5)],[(3,4)]])`
`SymmetricGroup(`$n$`)`, `AlternatingGroup(`$n$`)`
Abelian groups: `AbelianGroup([3,15])`
Matrix groups: `GL`, `SL`, `Sp`, `SU`, `GU`, `SO`, `GO`
Functions: `G.sylow_subgroup(p)`, `G.character_table()`,
`G.normal_subgroups()`, `G.cayley_graph()`

## Noncommutative rings

Quaternions: `Q.<i,j,k> = QuaternionAlgebra(a,b)`
Free algebra: `R.<a,b,c> = FreeAlgebra(QQ, 3)`

## Python modules

`import` *module_name*
`module_name.`⟨tab⟩ and `help(module_name)`

## Profiling and debugging

`time` *command*: show timing information
`timeit("command")`: accurately time command
`t = cputime(); cputime(t)`: elapsed CPU time
`t = walltime(); walltime(t)`: elapsed wall time
`%pdb`: turn on interactive debugger (command line only)
`%prun command`: profile command (command line only)