Sage Quick Reference

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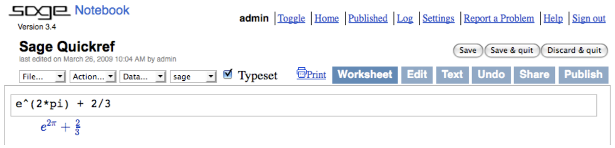
Notebook



セルの評価: ⟨shift-enter⟩

セルを評価し新しいセルを作る: ⟨alt-enter⟩

セルの分割: ⟨control-;⟩

セルの結合: ⟨control-backspace⟩

数式セルの挿入: セルの間の青い線をクリック

Text/HTML セルの挿入: セルの間の青い線を shift-click

セルの削除: 内容を削除したあとで backspace

Evaluate cell: ⟨shift-enter⟩
Evaluate cell creating new cell: ⟨alt-enter⟩
Split cell: ⟨control-;⟩
Join cells: ⟨control-backspace⟩
Insert math cell: click blue line between cells
Insert text/HTML cell: shift-click blue line between cells
Delete cell: delete content then backspace

Command line

com⟨tab⟩ で *command* を補完

`*`*bar*`*?` で "*bar*" を含むコマンド名をリストアップ

command`?`⟨tab⟩ でドキュメントを表示

command`??`⟨tab⟩ でソースコードを表示

`a.`⟨tab⟩ でオブジェクト a のメソッドを表示 (more: `dir(a)`)

`a._`⟨tab⟩ で a の hidden methods を表示

`search_doc("`*string or regexp*`")` ドキュメントの全文検索

`search_src("`*string or regexp*`")` ソースコードの検索

`_` は直前の出力

com⟨tab⟩ complete *command*
`*`*bar*`*?` list command names containing "*bar*"
command`?`⟨tab⟩ shows documentation
command`??`⟨tab⟩ shows source code
`a.`⟨tab⟩ shows methods for object a (more: `dir(a)`)
`a._`⟨tab⟩ shows hidden methods for object a
`search_doc("`*string or regexp*`")` fulltext search of docs
`search_src("`*string or regexp*`")` search source code
`_` is previous output

Numbers

整数: $\mathbb{Z}$ = `ZZ` 例 `-2 -1 0 1 10^100`

有理数: $\mathbb{Q}$ = `QQ` 例 `1/2 1/1000 314/100 -2/1`

実数: $\mathbb{R} \approx$ `RR` 例 `.5 0.001 3.14 1.23e10000`

複素数: $\mathbb{C} \approx$ `CC` 例 `CC(1,1) CC(2.5,-3)`

倍精度 (Double): `RDF` and `CDF` 例 `CDF(2.1,3)`

Mod n: $\mathbb{Z}/n\mathbb{Z}$ = `Zmod` 例 `Mod(2,3) Zmod(3)(2)`

有限体: $\mathbb{F}_q$ = `GF` 例 `GF(3)(2) GF(9,"a").0`

多項式: $R[x,y]$ 例 `S.<x,y>=QQ[] x+2*y^3`

巾級数: $R[[t]]$ 例 `S.<t>=QQ[[]] 1/2+2*t+O(t^2)`

p 進整数: $\mathbb{Z}_p \approx$`Zp`, $\mathbf{Q}_p \approx$`Qp` 例 `2+3*5+O(5^2)`

代数閉包: $\overline{\mathbb{Q}}$ = `QQbar` 例 `QQbar(2^(1/5))`

区間演算: `RIF` 例 `RIF((1,1.00001))`

数体: `R.<x>=QQ[]; K.<a>=NumberField(x^3+x+1)`

Integers: $\mathbb{Z}$ = `ZZ` e.g. `-2 -1 0 1 10^100`
Rationals: $\mathbb{Q}$ = `QQ` e.g. `1/2 1/1000 314/100 -2/1`
Reals: $\mathbb{R} \approx$ `RR` e.g. `.5 0.001 3.14 1.23e10000`
Complex: $\mathbb{C} \approx$ `CC` e.g. `CC(1,1) CC(2.5,-3)`
Double precision: `RDF` and `CDF` e.g. `CDF(2.1,3)`
Mod n: $\mathbb{Z}/n\mathbb{Z}$ = `Zmod` e.g. `Mod(2,3) Zmod(3)(2)`
Finite fields: $\mathbb{F}_q$ = `GF` e.g. `GF(3)(2) GF(9,"a").0`
Polynomials: $R[x,y]$ e.g. `S.<x,y>=QQ[] x+2*y^3`
Series: $R[[t]]$ e.g. `S.<t>=QQ[[]] 1/2+2*t+O(t^2)`
p-adic numbers: $\mathbb{Z}_p \approx$`Zp`, $\mathbf{Q}_p \approx$`Qp` e.g. `2+3*5+O(5^2)`
Algebraic closure: $\overline{\mathbb{Q}}$ = `QQbar` e.g. `QQbar(2^(1/5))`
Interval arithmetic: `RIF` e.g. `RIF((1,1.00001))`
Number field: `R.<x>=QQ[]; K.<a>=NumberField(x^3+x+1)`

Arithmetic

ab = `a*b` $\frac{a}{b}$ = `a/b` a^b = `a^b` $\sqrt{x}$ = `sqrt(x)`

$\sqrt[n]{x}$ = `x^(1/n)` $|x|$ = `abs(x)` $\log_b(x)$ = `og(x,b)`

和: $\sum_{i=k}^{n} f(i)$ = `sum(f(i) for i in (k..n))`

積: $\prod_{i=k}^{n} f(i)$ = `prod(f(i) for i in (k..n))`

ab = `a*b` $\frac{a}{b}$ = `a/b` a^b = `a^b` $\sqrt{x}$ = `sqrt(x)`
$\sqrt[n]{x}$ = `x^(1/n)` $|x|$ = `abs(x)` $\log_b(x)$ = `og(x,b)`
Sums: $\sum_{i=k}^{n} f(i)$ = `sum(f(i) for i in (k..n))`
Products: $\prod_{i=k}^{n} f(i)$ = `prod(f(i) for i in (k..n))`

Constants and functions

定数: π = `pi` e = `e` i = `i` ∞ = `oo`

ϕ = `golden_ratio` γ = `euler_gamma`

近似値: `pi.n(digits=18)` = 3.14159265358979324

関数: `sin cos tan sec csc cot sinh cosh tanh sech csch coth log ln exp ...`

Python の関数: `def f(x): return x^2`

Constants: π = `pi` e = `e` i = `i` ∞ = `oo`
ϕ = `golden_ratio` γ = `euler_gamma`
Approximate: `pi.n(digits=18)` = 3.14159265358979324
Functions: `sin cos tan sec csc cot sinh cosh tanh sech csch coth log ln exp ...`
Python function: `def f(x): return x^2`

Interactive functions

関数の前に `@interact` を置く (変数で controls が決まる)

```
@interact
def f(n=[0..4], s=(1..5), c=Color("red")):
    var("x")
    show(plot(sin(n+x^s),-pi,pi,color=c))
```

Put `@interact` before function (vars determine controls)

```
@interact
def f(n=[0..4], s=(1..5), c=Color("red")):
    var("x")
    show(plot(sin(n+x^s),-pi,pi,color=c))
```

Symbolic expressions

新しい不定元 (symbolic variables) を定義: `var("t u v y z")`

Symbolic function: 例 $f(x) = x^2$ `f(x)=x^2`

関係式: `f==g f<=g f>=g f<g f>g`

$f = g$ を解く: `solve(f(x)==g(x), x)`
`solve([f(x,y)==0, g(x,y)==0], x,y)`

`factor(...)` `expand(...)` `(...).simplify_...`

$x \in [a,b]$ s.t. $f(x) \approx 0$ を見付ける: `find_root(f(x), a, b)`

Define new symbolic variables: `var("t u v y z")`
Symbolic function: e.g. $f(x) = x^2$ `f(x)=x^2`
Relations: `f==g f<=g f>=g f<g f>g`
Solve $f = g$: `solve(f(x)==g(x), x)`
`solve([f(x,y)==0, g(x,y)==0], x,y)`
`factor(...)` `expand(...)` `(...).simplify_...`
`find_root(f(x), a, b)` find $x \in [a,b]$ s.t. $f(x) \approx 0$

Calculus

$\lim_{x \to a} f(x)$ = `limit(f(x), x=a)`

$\frac{d}{dx}(f(x))$ = `diff(f(x),x)`

$\frac{\partial}{\partial x}(f(x,y))$ = `diff(f(x,y),x)`

`diff` = `differentiate` = `derivative`

$\int f(x)dx$ = `integral(f(x),x)`

$\int_a^b f(x)dx$ = `integral(f(x),x,a,b)`

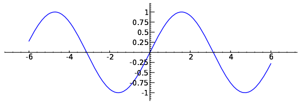
$\int_a^b f(x)dx \approx$ `numerical_integral(f(x),a,b)`

a に関する次数 n の Taylor 多項式: `taylor(f(x),x,`a`,`n`)`

$\lim_{x \to a} f(x)$ = `limit(f(x), x=a)`
$\frac{d}{dx}(f(x))$ = `diff(f(x),x)`
$\frac{\partial}{\partial x}(f(x,y))$ = `diff(f(x,y),x)`
`diff` = `differentiate` = `derivative`
$\int f(x)dx$ = `integral(f(x),x)`
$\int_a^b f(x)dx$ = `integral(f(x),x,a,b)`
$\int_a^b f(x)dx \approx$ `numerical_integral(f(x),a,b)`

Taylor polynomial, deg n about a: taylor(f(x),x,a,n)

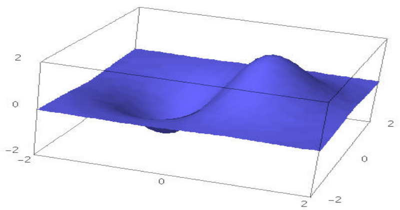
2D graphics



line([(x_1,y_1),...,(x_n,y_n)],*options*)
polygon([(x_1,y_1),...,(x_n,y_n)],*options*)
circle((x,y),r,*options*)
text("txt",(x,y),*options*)
options は plot.options にあるもの,
例 thickness=*pixel*, rgbcolor=(r,g,b), hue=h
ただし $0 \le r, b, g, h \le 1$
show(*graphic*, *options*)
サイズの調整には figsize=[w,h] を使う
縦横比を調整するには aspect_ratio=*number* を使う
plot(f(x),(x,$x_{\min}$,$x_{\max}$),*options*)
parametric_plot((f(t),g(t)),(t,$t_{\min}$,$t_{\max}$),*options*)
polar_plot(f(t),(t,$t_{\min}$,$t_{\max}$),*options*)
結合: circle((1,1),1)+line([(0,0),(2,2)])
animate(*list of graphics*, *options*).show(delay=20)

line([(x_1,y_1),...,(x_n,y_n)],*options*)
polygon([(x_1,y_1),...,(x_n,y_n)],*options*)
circle((x,y),r,*options*)
text("txt",(x,y),*options*)
options as in plot.options,
e.g. thickness=*pixel*, rgbcolor=(r,g,b), hue=h
where $0 \le r, b, g, h \le 1$
show(*graphic*, *options*)
use figsize=[w,h] to adjust size
use aspect_ratio=*number* to adjust aspect ratio
plot(f(x),(x,$x_{\min}$,$x_{\max}$),*options*)
parametric_plot((f(t),g(t)),(t,$t_{\min}$,$t_{\max}$),*options*)
polar_plot(f(t),(t,$t_{\min}$,$t_{\max}$),*options*)
combine: circle((1,1),1)+line([(0,0),(2,2)])
animate(*list of graphics*, *options*).show(delay=20)

3D graphics



line3d([(x_1,y_1,z_1),...,(x_n,y_n,z_n)],*options*)
sphere((x,y,z),r,*options*)
text3d("txt", (x,y,z), *options*)
tetrahedron((x,y,z),*size*,*options*)
cube((x,y,z),*size*,*options*)
octahedron((x,y,z),*size*,*options*)
dodecahedron((x,y,z),*size*,*options*)
icosahedron((x,y,z),*size*,*options*)
plot3d(f(x,y),(x,x_b,x_e), (y,y_b,y_e),*options*)
parametric_plot3d((f,g,h),(t,t_b,t_e),*options*)
parametric_plot3d((f(u,v),g(u,v),h(u,v)),
(u,u_b,u_e),(v,v_b,v_e),*options*)
options: aspect_ratio=[1,1,1], color="red",
opacity=0.5, figsize=6, viewer="tachyon"

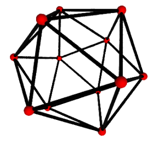
line3d([(x_1,y_1,z_1),...,(x_n,y_n,z_n)],*options*)
sphere((x,y,z),r,*options*)
text3d("txt", (x,y,z), *options*)
tetrahedron((x,y,z),*size*,*options*)
cube((x,y,z),*size*,*options*)
octahedron((x,y,z),*size*,*options*)
dodecahedron((x,y,z),*size*,*options*)
icosahedron((x,y,z),*size*,*options*)
plot3d(f(x,y),(x,x_b,x_e), (y,y_b,y_e),*options*)
parametric_plot3d((f,g,h),(t,t_b,t_e),*options*)
parametric_plot3d((f(u,v),g(u,v),h(u,v)),
(u,u_b,u_e),(v,v_b,v_e),*options*)
options: aspect_ratio=[1,1,1], color="red",
opacity=0.5, figsize=6, viewer="tachyon"

Discrete math

$\lfloor x \rfloor$ = floor(x) $\lceil x \rceil$ = ceil(x)
n を k で割った余り = n%k $k|n$ iff n%k==0
$n!$ = factorial(n) $\binom{x}{m}$ = binomial(x,m)
$\phi(n)$ = euler_phi(n)
文字列 (String): 例 s = "Hello" = "He"+'llo'
s[0]="H" s[-1]="o" s[1:3]="el" s[3:]="lo"
リスト (List): 例 [1,"Hello",x] = []+[1,"Hello"]+[x]
タプル (Tuple): 例 (1,"Hello",x) (immutable)
集合 (Set): 例 $\{1,2,1,a\}$ = Set([1,2,1,"a"]) ($=\{1,2,a\}$)
集合の内包的記法 ≈ リストの内包表記, 例
$\{f(x)|x \in X, x > 0\}$ =Set([f(x) for x in X if x>0])

$\lfloor x \rfloor$ = floor(x) $\lceil x \rceil$ = ceil(x)
Remainder of n divided by k = n%k $k|n$ iff n%k==0
$n!$ = factorial(n) $\binom{x}{m}$ = binomial(x,m)
$\phi(n)$ = euler_phi(n)
Strings: e.g. s = "Hello" = "He"+'llo'
s[0]="H" s[-1]="o" s[1:3]="el" s[3:]="lo"
Lists: e.g. [1,"Hello",x] = []+[1,"Hello"]+[x]
Tuples: e.g. (1,"Hello",x) (immutable)
Sets: e.g. $\{1,2,1,a\}$ = Set([1,2,1,"a"]) ($=\{1,2,a\}$)
List comprehension ≈ set builder notation, e.g.
$\{f(x)|x \in X, x > 0\}$ =Set([f(x) for x in X if x>0])

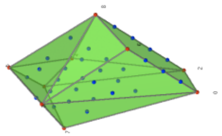
Graph theory



グラフ: G = Graph({0:[1,2,3], 2:[4]})
有向グラフ: DiGraph(*dictionary*)
グラフの族: graphs.⟨tab⟩
不変量: G.chromatic_polynomial(), G.is_planar()
パス: G.shortest_path()
可視化: G.plot(), G.plot3d()
自己同型: G.automorphism_group(),
G1.is_isomorphic(G2), G1.is_subgraph(G2)

Graph: G = Graph({0:[1,2,3], 2:[4]})
Directed Graph: DiGraph(*dictionary*)
Graph families: graphs.⟨tab⟩
Invariants: G.chromatic_polynomial(), G.is_planar()
Paths: G.shortest_path()
Visualize: G.plot(), G.plot3d()
Automorphisms: G.automorphism_group(),
G1.is_isomorphic(G2), G1.is_subgraph(G2)

Combinatorics



整数列: sloane_find(*list*), sloane.⟨tab⟩
分割: P=Partitions(n) P.count()
組合せ (部分リスト): C=Combinations(*list*) C.list()
直積: CartesianProduct(P,C)
ヤング盤 (Tableau): Tableau([[1,2,3],[4,5]])
ワード: W=Words("abc"); W("aabca")
半順序集合 (poset): Poset([[1,2],[4],[3],[4],[]])
ルート系: RootSystem(["A",3])
クリスタル: CrystalOfTableaux(["A",3], shape=[3,2])
Lattice Polytopes: A=random_matrix(ZZ,3,6,x=7)
L=LatticePolytope(A) L.npoints() L.plot3d()

Integer sequences: sloane_find(*list*), sloane.⟨tab⟩
Partitions: P=Partitions(n) P.count()
Combinations: C=Combinations(*list*) C.list()
Cartesian product: CartesianProduct(P,C)
Tableau: Tableau([[1,2,3],[4,5]])
Words: W=Words("abc"); W("aabca")
Posets: Poset([[1,2],[4],[3],[4],[]])
Root systems: RootSystem(["A",3])

Crystals: `CrystalOfTableaux(["A",3], shape=[3,2])`
Lattice Polytopes: `A=random_matrix(ZZ,3,6,x=7)`
`L=LatticePolytope(A) L.npoints() L.plot3d()`

Matrix algebra

$\begin{pmatrix}1\\2\end{pmatrix}$ = `vector([1,2])`

$\begin{pmatrix}1&2\\3&4\end{pmatrix}$ = `matrix(QQ,[[1,2],[3,4]], sparse=False)`

$\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}$ = `matrix(QQ,2,3,[1,2,3, 4,5,6])`

$\begin{vmatrix}1&2\\3&4\end{vmatrix}$ = `det(matrix(QQ,[[1,2],[3,4]]))`

Av = `A*v` A^{-1} = `A^-1` A^t = `A.transpose()`

$Ax = v$ を解く: `A\v` or `A.solve_right(v)`

$xA = v$ を解く: `A.solve_left(v)`

被約行階段行列: `A.echelon_form()`

階数と退化: `A.rank() A.nullity()`

Hessenberg 型: `A.hessenberg_form()`

特性多項式: `A.charpoly()`

固有値: `A.eigenvalues()`

固有ベクトル: `A.eigenvectors_right()` (also left)

Gram-Schmidt: `A.gram_schmidt()`

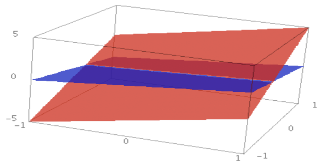
可視化: `A.plot()`

LLL reduction: `matrix(ZZ,...).LLL()`

Hermite 形式: `matrix(ZZ,...).hermite_form()`

$\begin{pmatrix}1\\2\end{pmatrix}$ = `vector([1,2])`
$\begin{pmatrix}1&2\\3&4\end{pmatrix}$ = `matrix(QQ,[[1,2],[3,4]], sparse=False)`
$\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}$ = `matrix(QQ,2,3,[1,2,3, 4,5,6])`
$\begin{vmatrix}1&2\\3&4\end{vmatrix}$ = `det(matrix(QQ,[[1,2],[3,4]]))`
Av = `A*v` A^{-1} = `A^-1` A^t = `A.transpose()`
Solve $Ax = v$: `A\v` or `A.solve_right(v)`
Solve $xA = v$: `A.solve_left(v)`
Reduced row echelon form: `A.echelon_form()`
Rank and nullity: `A.rank() A.nullity()`
Hessenberg form: `A.hessenberg_form()`
Characteristic polynomial: `A.charpoly()`
Eigenvalues: `A.eigenvalues()`
Eigenvectors: `A.eigenvectors_right()` (also left)
Gram-Schmidt: `A.gram_schmidt()`
Visualize: `A.plot()`
LLL reduction: `matrix(ZZ,...).LLL()`
Hermite form: `matrix(ZZ,...).hermite_form()`

Linear algebra



ベクトル空間 K^n = `K^n` 例 `QQ^3 RR^2 CC^4`

部分空間: `span(vectors, field)`
例 `span([[1,2,3], [2,3,5]], QQ)`

Kernel: `A.right_kernel()` (`left_` も)

和と共通部分: `V + W` と `V.intersection(W)`

基底: `V.basis()`

基底行列: `V.basis_matrix()`

行列を部分空間への制限: `A.restrict(V)`

基底を使ったベクトルの表示: `V.coordinates(vector)`

Vector space K^n = `K^n` e.g. `QQ^3 RR^2 CC^4`
Subspace: `span(vectors, field)`
E.g., `span([[1,2,3], [2,3,5]], QQ)`
Kernel: `A.right_kernel()` (also left)
Sum and intersection: `V + W` and `V.intersection(W)`
Basis: `V.basis()`
Basis matrix: `V.basis_matrix()`
Restrict matrix to subspace: `A.restrict(V)`
Vector in terms of basis: `V.coordinates(vector)`

Numerical mathematics

パッケージ: `import numpy, scipy, cvxopt`

最小化: `var("x y z")`
`minimize(x^2+x*y^3+(1-z)^2-1, [1,1,1])`

Packages: `import numpy, scipy, cvxopt`
Minimization: `var("x y z")`
`minimize(x^2+x*y^3+(1-z)^2-1, [1,1,1])`

Number theory

素数: `prime_range(n,m)`, `is_prime`, `next_prime`

素因数分解: `factor(n)`, `qsieve(n)`, `ecm.factor(n)`

Kronecker symbol: $\left(\frac{a}{b}\right)$ = `kronecker_symbol(a,b)`

連分数: `continued_fraction(x)`

Bernoulli 数: `bernoulli(n)`, `bernoulli_mod_p(p)`

楕円曲線: `EllipticCurve(`$[a_1, a_2, a_3, a_4, a_6]$`)`

Dirichlet characters: `DirichletGroup(N)`

Modular forms: `ModularForms(level, weight)`

Modular symbols: `ModularSymbols(level,weight,sign)`

Brandt modules: `BrandtModule(level, weight)`

Modular abelian varieties: `J0(N)`, `J1(N)`

Primes: `prime_range(n,m)`, `is_prime`, `next_prime`
Factor: `factor(n)`, `qsieve(n)`, `ecm.factor(n)`
Kronecker symbol: $\left(\frac{a}{b}\right)$ = `kronecker_symbol(a,b)`
Continued fractions: `continued_fraction(x)`
Bernoulli numbers: `bernoulli(n)`, `bernoulli_mod_p(p)`
Elliptic curves: `EllipticCurve(`$[a_1, a_2, a_3, a_4, a_6]$`)`
Dirichlet characters: `DirichletGroup(N)`
Modular forms: `ModularForms(level, weight)`
Modular symbols: `ModularSymbols(level,weight,sign)`
Brandt modules: `BrandtModule(level, weight)`
Modular abelian varieties: `J0(N)`, `J1(N)`

Group theory

`G = PermutationGroup([[(1,2,3),(4,5)],[(3,4)]])`

`SymmetricGroup(n)`, `AlternatingGroup(n)`

アーベル群: `AbelianGroup([3,15])`

行列群: `GL`, `SL`, `Sp`, `SU`, `GU`, `SO`, `GO`

関数: `G.sylow_subgroup(p)`, `G.character_table()`,
`G.normal_subgroups()`, `G.cayley_graph()`

`G = PermutationGroup([[(1,2,3),(4,5)],[(3,4)]])`
`SymmetricGroup(n)`, `AlternatingGroup(n)`
Abelian groups: `AbelianGroup([3,15])`
Matrix groups: `GL`, `SL`, `Sp`, `SU`, `GU`, `SO`, `GO`
Functions: `G.sylow_subgroup(p)`, `G.character_table()`,
`G.normal_subgroups()`, `G.cayley_graph()`

Noncommutative rings

四元数: `Q.<i,j,k> = QuaternionAlgebra(a,b)`

自由代数: `R.<a,b,c> = FreeAlgebra(QQ, 3)`

Quaternions: `Q.<i,j,k> = QuaternionAlgebra(a,b)`
Free algebra: `R.<a,b,c> = FreeAlgebra(QQ, 3)`

Python modules

`import module_name`

`module_name.`⟨tab⟩ and `help(module_name)`

`import module_name`
`module_name.`⟨tab⟩ and `help(module_name)`

Profiling and debugging

`time command`: timing information の表示

`timeit("command")`: accurately time command

`t = cputime(); cputime(t)`: 経過した CPU time

`t = walltime(); walltime(t)`: 経過した wall time

`%pdb`: interactive debugger を開始 (command line only)

`%prun command`: profile command (command line only)

`time command`: show timing information
`timeit("command")`: accurately time command
`t = cputime(); cputime(t)`: elapsed CPU time
`t = walltime(); walltime(t)`: elapsed wall time
`%pdb`: turn on interactive debugger (command line only)
`%prun command`: profile command (command line only)